ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

COURSE MATERIAL

COURSE NAME: BATTERY MANAGEMENT SYSTEM

UNIT 1
INTRODUCTION

Battery Boot Camp:

Battery management system (BMS) is technology dedicated to the oversight of a battery pack, which is an assembly of battery cells, electrically organized in a row x column matrix configuration to enable delivery of targeted range of voltage and current for a duration of time against expected load scenarios.

How Lithium ion cell works?

The oversight that a BMS provides usually includes:

- Monitoring the battery
- Providing battery protection
- Estimating the battery's operational state
- Continually optimizing battery performance
- Reporting operational status to external devices

Here, the term "battery" implies the entire pack; however, the monitoring and control functions are specifically applied to individual cells, or groups of cells called modules in the overall battery pack assembly. Lithium-ion rechargeable cells have the highest energy density and are the standard choice for battery packs for many consumer products, from laptops to electric vehicles. While they perform superbly, they can be rather unforgiving if operated outside a generally tight safe operating area (SOA), with outcomes ranging from compromising the battery performance to outright dangerous consequences. The BMS certainly has a challenging job description, and its overall complexity and oversight outreach may span many disciplines such as electrical, digital, control, thermal, and hydraulic.

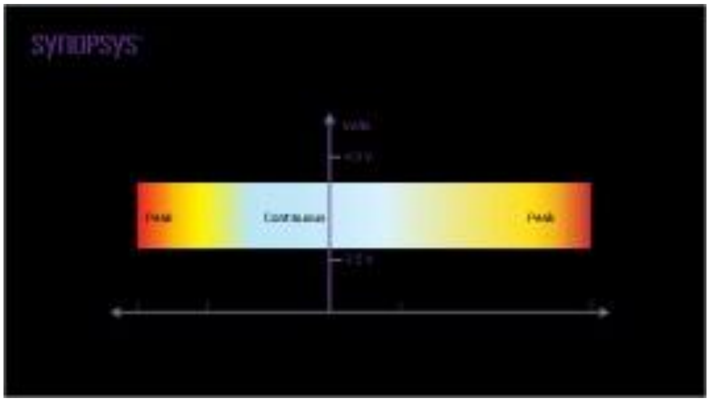
Battery management systems do not have a fixed or unique set of criteria that must be adopted. The technology design scope and implemented features generally correlate with:

- The costs, complexity, and size of the battery pack
- Application of the battery and any safety, lifespan, and warranty concerns
- Certification requirements from various government regulations where costs and penalties are paramount if inadequate functional safety measures are in place

There are many BMS design features, with battery pack protection management and capacity management being two essential features. We'll discuss how these two features work here. Battery pack protection management has two key arenas: electrical protection, which implies not allowing the battery to be damaged via usage outside its SOA, and thermal protection, which involves passive and/or active temperature control to maintain or bring the pack into its SOA.

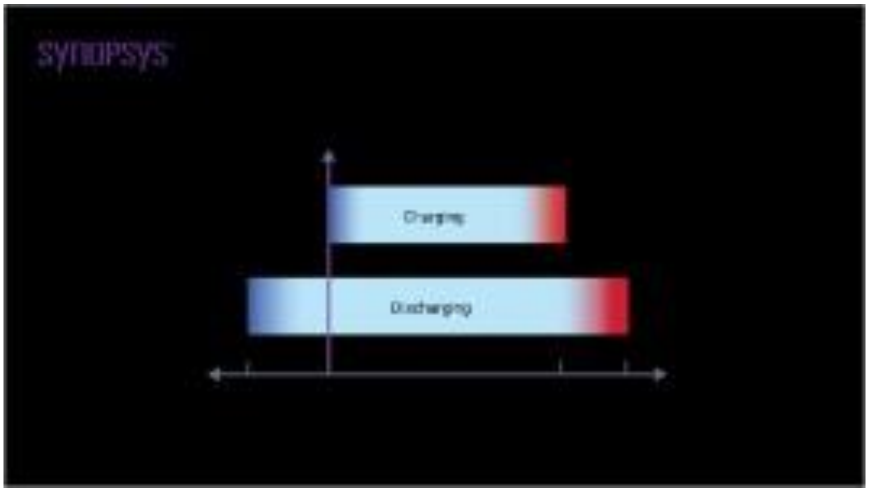
Electrical Management Protection: Current

Monitoring battery pack current and cell or module voltages is the road to electrical protection. The electrical SOA of any battery cell is bound by current and voltage. Figure 1 illustrates a typical lithium-ion cell SOA, and a well-designed BMS will protect the pack by preventing operation outside the manufacturer's cell ratings. In many cases, further derating may be applied to reside within the SOA safe zone in the interest of promoting further battery lifespan.



BMS sensing and high-voltage control:

lithium-ion cell must operate within a certain voltage range. These SOA boundaries will ultimately be determined by the intrinsic chemistry of the selected lithium-ion cell and the temperature of the cells at any given time. Moreover, since any battery pack experiences a significant amount of current cycling, discharging due to load demands and charging from a variety of energy sources, these SOA voltage limits are usually further constrained to optimize battery lifespan. The BMS must know what these limits are and will command decisions based upon the proximity to these thresholds. For example, when approaching the high voltage limit, a BMS may request a gradual reduction of charging current, or may request the charging current be terminated altogether if the limit is reached. However, this limit is usually accompanied by additional intrinsic voltage hysteresis considerations to prevent control chatter about the shutdown threshold. On the other hand, when approaching the low voltage limit, a BMS will request that key active offending loads reduce their current demands. In the case of an electric vehicle, this may be carried out by reducing the allowed torque available to the traction motor. Of course, the BMS must make safety considerations for the driver the highest priority while protecting the battery pack to prevent permanent damage



HIGH-VOLTAGE BMS FEATURES

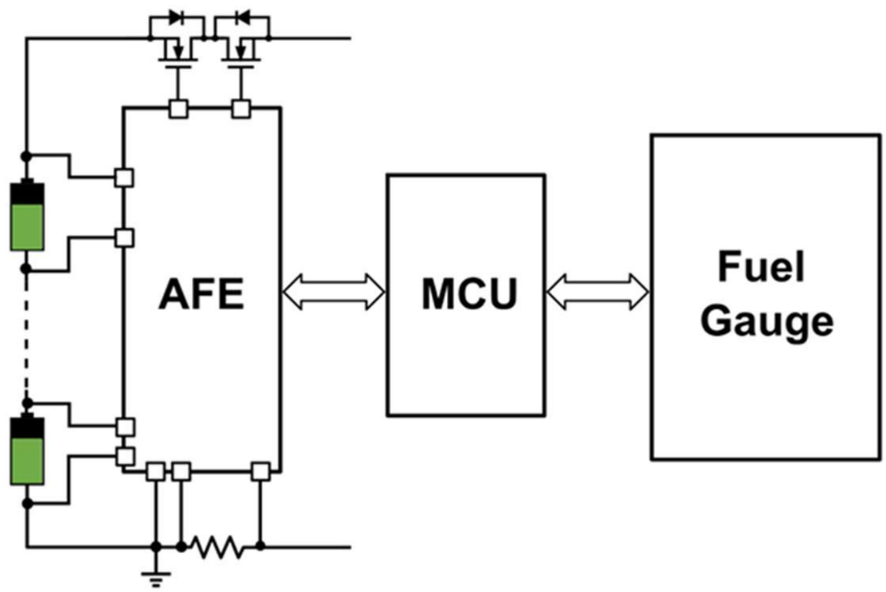
- From kWh to MWh, the Nuvation Energy High-Voltage BMS manages up to 1250 VDC per battery stack and up to 36 stacks in parallel with the addition of a Multi Stack Controller.
- Connects and disconnects a battery stack to the DC Bus of the ESS in response to requests from system controllers. Will also report the resulting capacity change to the PCS and other energy management systems.
- Measures cell- and stack-level voltage, temperature, and current. Calculates State of Charge (SOC), and Depth of Discharge (DOD) and provides this information to the PCS to enable precise charge/discharge control.
- Provides battery profile thresholds to the PCS, triggers warnings when approaching safety limits, can activate battery cooling fans.
- Will alter current limits to protect the battery from overcharge and over-discharge as well as to reduce battery temperature and to prevent over-voltage.

- Utilizes a pre-charge circuit when connecting the stack to the DC bus, to prevent current surges and voltage mismatches that could damage cells.
- Disconnects batteries from the power path if safety thresholds are exceeded during ESS operation.
- Includes short-circuit protection.

BMS design requirements:

Battery-powered applications have become commonplace over the last decade, and such devices require a certain level of protection to ensure safe usage. The battery management system (BMS) monitors the battery and possible fault conditions, preventing the battery from situations in which it can degrade, fade in capacity, or even potentially harm the user or surrounding environment. It is also the responsibility of the BMS to provide an accurate state-of-charge (SOC) and state-of-health (SOH) estimate to ensure an informative and safe user experience over the lifetime of the battery. Designing a proper BMS is critical not only from a safety point of view, but also for customer satisfaction.

The main structure of a complete BMS for low or medium voltages is commonly made up of three ICs: an analog front-end (AFE), a microcontroller (MCU), and a fuel gauge (see Figure 1). The fuel gauge can be a standalone IC, or it can be embedded in the MCU. The MCU is the central element of the BMS, taking information from both the AFE and fuel gauge and interfacing with the rest of the system.



The AFE provides the MCU and fuel gauge with voltage, temperature, and current readings from the battery. Since the AFE is physically closest to the battery, it is recommended that the AFE also controls the circuit breakers, which disconnect the battery from the rest of the system if any faults are triggered.

The fuel gauge IC takes the readings from the AFE, then uses complex cell modeling and advanced algorithms to estimate key parameters, such as the state-of-charge (SOC) and state-of-health (SOH). Similar to the AFE, some of the fuel gauge's tasks can be included in the MCU code; however, using a dedicated fuel gauge IC, such as MPS's MPF4279x fuel gauge family, offers several advantages:

- Efficient design: Using dedicated ICs to run complex fuel gauge algorithms allows designers to use MCUs with lower specifications, reducing overall cost and current consumption.

- Improved insights and safety: A dedicated fuel gauge can measure the individual SOC and SOH of each series cell combination in the battery pack, which enables more precise measurement accuracy and aging detection over the lifespan of the battery. This is important because cell impedances and capacities can diverge over time, leading to run-time and safety implications.
- Fast time-to-market: Fuel gauge ICs come fully tested for a variety of situations and test cases. This reduces the time and cost of testing complex algorithms, while simultaneously enabling faster time-to-market.

How are cells made? How can they fail?

Benjamin Franklin's famous experiment to attract electricity by flying a kite in a lightning storm was only one of many late eighteenth- and early nineteenth-century experiments conducted to learn about electricity. The first battery was constructed in 1800 by Italian Alessandro Volta. The so-called *voltaic pile* consisted of alternating discs of silver and zinc separated by leather or pasteboard that had been soaked in salt water, lye, or some alkaline solution. Strips of metal at each end of the pile were connected to small cups filled with mercury. When Volta touched both cups of mercury with his fingers, he received an electric shock; the more discs he assembled, the greater the jolt he received.

Volta's discovery led to further experimentation. In 1813, Sir Humphrey Davy constructed a pile with 2,000 pairs of discs in the basement of the Royal Institution of London. Among other applications, Davy used the electricity he produced for electrolysis—catalyzing chemical reactions by passing a current through substances (Davy separated sodium and potassium from compounds). Only a few years later, Michael Faraday discovered the principle of electromagnetic induction, using a magnet to induce electricity in a coiled wire. This technique is at the heart of the dynamos used to produce electricity in power plants today. (While a dynamo produces alternating current (AC) in which the flow of electricity shifts direction regularly, batteries produce direct current (DC) that flows in one direction only.) A lead-acid cell capable of producing a very large amount of current, the forerunner of today's **automobile** battery, was devised in 1859 by Frenchman Gaston Planté.

In the United States, Thomas Edison was experimenting with electricity from both batteries and dynamos to power the light bulb, which began to spread in the United States in the early 1880s. During the 1860s, Georges Leclanché invented the wet cell, which, though heavy because of its liquid components, could be sold and used commercially. By the 1870s and 1880s, the Leclanché cell was being produced using dry materials and was used for a number of tasks, including providing power for Alexander Graham Bell's telephone and for the newly-invented flashlight. Batteries were subsequently called upon to provide power for many other inventions, such as the radio, which became hugely popular in the years following World War I. Today, more than twenty billion power cells are sold throughout the world each year, and each American uses approximately 27 batteries annually.

Design

All batteries utilize similar procedures to create electricity; however, variations in materials and construction have produced different types of batteries. Strictly speaking, what is commonly termed a battery is actually a group of linked cells. The following is a simplified description of how a battery works.

Two important parts of any cell are the anode and the cathode. The cathode is a metal that is combined, naturally or in the laboratory, with oxygen—the combination is called an *oxide.* Iron oxid (rust), although too fragile to use in a battery, is perhaps the most familiar oxide. Some other oxides are actually strong enough to be worked (cut, bent, shaped, molded, and so on) and used in a cell. The anode is a metal that would oxidize if it were allowed to and, other things being equal, is more likely to oxidize than the metal that forms part of the cathode.

A cell produces electricity when one end of a cathode and one end of an anode are placed into a third substance that can conduct electricity, while their other ends are connected. The anode draws oxygen atoms toward it, thereby creating an electric flow. If there is a switch in the circuit (similar to any wall or lamp switch), the circuit is not complete and electricity cannot flow unless the switch is in the closed position. If, in addition to the switch, there is something else in the circuit, such as a light bulb, the bulb will light from the friction of the electrons moving through it.

The third substance into which the anode and the cathode are placed is called an *electrolyte*. In many cases this material is a chemical combination that has the property of being alkaline. Thus, an alkaline battery is one that makes use of an alkaline electrolyte. A cell will not produce electricity by itself unless it is placed in a circuit that has been rendered complete by a simple switch, or by some other switching connection in the appliance using the battery.

Designing a cell can lead to many variations in type and structure. Not all electrolytes, for example, are alkaline. Additionally, the container for the electrolyte can act as both a container and either the cathode or the anode. Some cells draw their oxygen not from a cathode but right out of the air. Changes in the compositions of the anode and the cathode will provide more or less electricity. Precise adjustment of all of the materials used in a cell can affect the amount of electricity that can be produced, the rate of production, the voltage at which electricity is delivered through the lifetime of the cell, and the cell's ability to function at different temperatures.

All of these possibilities do, in fact, exist, and their various applications have produced the many different types of batteries available today (lithium, mercury, and so on). For years, however, the most common cell has been the 1.5 volt alkaline battery.

Different batteries function better in different circumstances. The alkaline 1.5 volt cell is ideal for photographic equipment, handheld computers and calculators, toys, tape recorders, and other "high drain" uses; it is also good in low temperatures. This cell has a sloping discharge characteristic—it loses power gradually, rather than ceasing to produce electricity suddenly—and will lose perhaps four percent of its power per year if left unused on a shelf.

Other types of batteries include a lithium/manganese dioxide battery, which has a flat discharge characteristic—it provides approximately the same amount of power at the beginning of its life as at the end—and can be used where there is a need for small, high-power batteries (smoke alarms, cameras, memory backups on computers, and so on). Hearing aids, pagers, and some other types of medical equipment frequently use zinc air button type batteries, which provide a high energy density on continuous discharge. A mercury battery is frequently used in many of the same applications as the zinc air battery, because it, too, provides a steady output voltage.

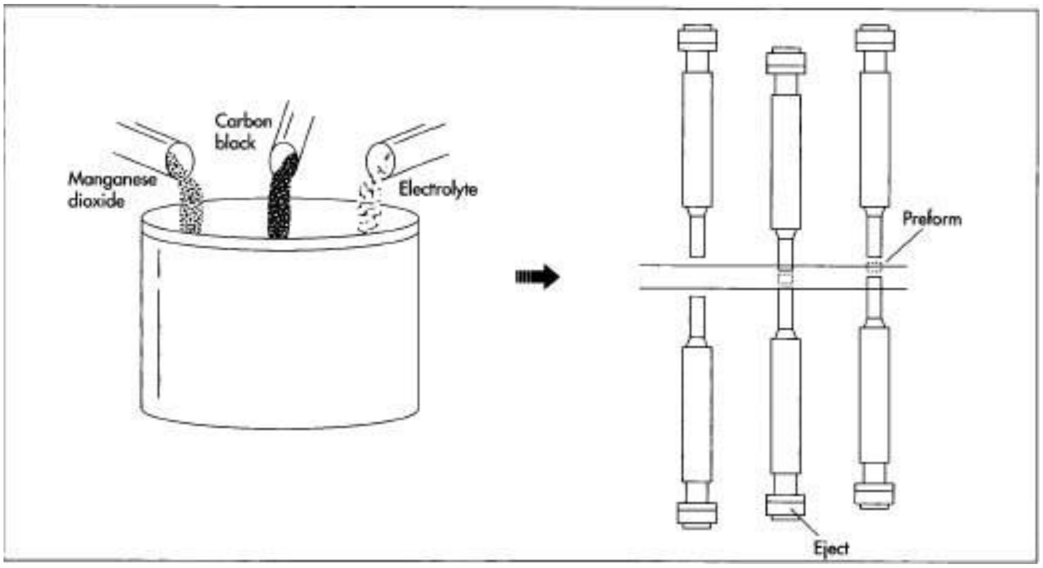
Raw Materials

This section, as well as the following section, will focus on alkaline batteries. In an alkaline battery, the cylinder that contains the cells is made of nickel-plated steel. It is lined with a separator that divides the cathode from the anode and is made of either layered paper or a porous synthetic material. The canister is sealed at one end with an asphalt or epoxy sealant that underlies a steel plate, and at the other with a brass nail driven through the cylinder. This nail is welded to a metal end cap and passed through an exterior plastic seal. Inside the cylinder, the cathode consists of a mixture of manganese dioxide, graphite, and a potassium hydroxide solution; the anode comprises zinc powder and a potassium hydroxide electrolyte.

The Manufacturing Process

The cathode

- 1 In an alkaline battery, the cathode actually doubles as part of the container. Huge loads of the constituent ingredients—manganese dioxide, carbon black (graphite), and an electrolyte (potassium hydroxide in solution)—are



Mixing the constituent ingredients is the first step in battery manufacture. After granulation, the mixture is then pressed or compacted into preforms—hollow cylinders. The principle involved in compaction is simple: a steel punch descends into a cavity and compacts the mixture. As it retracts, a punch from below rises to eject the compacted preform.

delivered by train and mixed in very large batches at the production site. The mixture is then granulated and pressed or compacted into hollow cylinders called *preforms*. Depending on the size of the battery being made, several preforms may be stacked one on top of another in a battery. Alternatively, the series of preforms can be replaced by an extruded ring of the same material.

- 2 The preforms are next inserted into a nickel-plated steel can; the combination of the preforms and the steel can make up the cathode of the battery. In a large operation, the cans are made at the battery factory using standard cutting and forming techniques. An indentation is made near the top of the can, and an asphalt or epoxy sealant is placed above the indentation to protect against leakage.

The separator

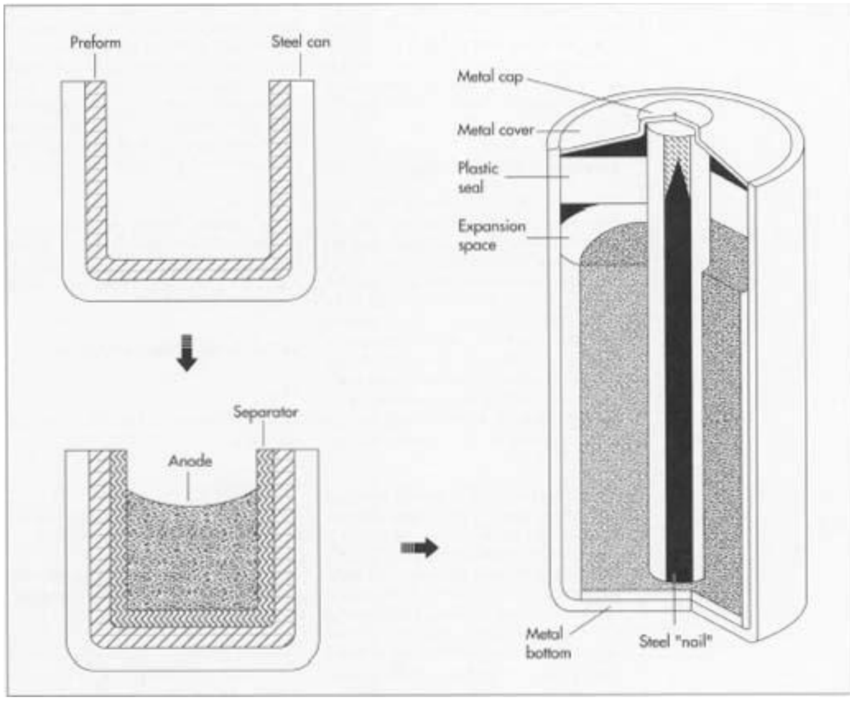
- 3 A paper separator soaked in the electrolyte solution is then inserted inside the can against the preforms; the separator is made from several pieces of paper laid at crossgrains to each other (like plywood). Looking down at an open can, one would see what looks like a paper cup inserted into the can. The separator keeps the cathode material from coming into contact with the anode material. As an alternative, a manufacturer might use a porous synthetic fiber for the same purpose.

The anode

- 4 The anode goes into the battery can next. It is a gel composed primarily of zinc powder, along with other materials including a potassium hydroxide electrolyte. This gel has the consistency of a very thick paste. Rather than a solution, it is chemically a suspension, in which particles do not settle (though an appropriate filter could separate them). The gel does not fill the can to the top so as to allow space for the chemical reactions that will occur once the battery is put into use.

The seals

- 5 Though the battery is able to produce electricity at this point, an open cell is not practical and would exhaust its potential rapidly. The battery needs to be sealed with three connected components. The first, a brass "nail" or long spike, is inserted into the middle of the can, through the gel material and serves as a "current collector." The second is a plastic seal and the third a metal end cap. The nail, which extends about two-thirds



The container of a typical alkaline battery, consisting of preform inserted into a steel can, also doubles as the cathode. The anode in the middle is a gel composed primarily of zinc powder. The separator between the anode and cathode is either paper or synthetic fiber that has been soaked in an electrolyte solution. In the finished battery, a plastic seal, a steel nail, and a metal top and bottom have been added. The nail is welded to the metal bottom and extends about two-thirds of the way into the can, through the anode.

of the way into the can, is welded to the metal end cap and then passed through the plastic seal.

- 6 This seal is significantly thinner in some places than in others, so that if too much gas builds up in the can, the seal will rupture rather than the entire battery. Some battery designs make use of a wax-filled hole in the plastic; excess gas pushes through the wax rather than rupturing the battery. The seal assembly meets the indentation made in the can at the beginning of the process and is crimped in place.
- 7 The opposite end of the can (the positive end of the battery) is then closed with a steel plate that is either welded in place or glued with an epoxy-type cement.

The label

- 8 Before the battery leaves the factory, a label is added identifying the type of battery, its size, and other information. The label is often paper that is simply glued to the battery. One large manufacturer has its label design printed on plastic shrink wrap: a loose fitting piece of heat-sensitive plastic is wrapped around the battery can and then exposed to a blast of heat that makes the plastic shrink down to fit tightly around the can.

UNIT II

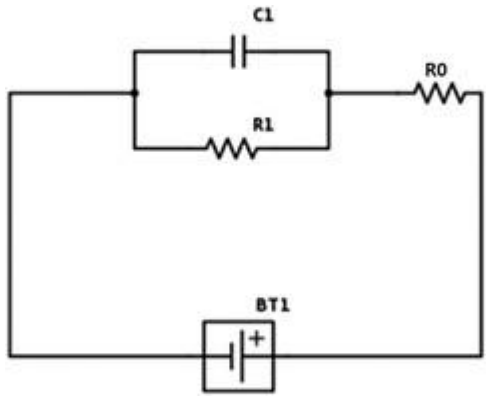
EQUIVALENT CIRCUIT CELL MODEL SIMULATION

Defining an equivalent-circuit model of a Li-ion cell:

This study investigates lithium-ion (Li-ion) battery discharge at a constant current by comparing equivalent circuit simulation data with experimental data. The simulations employ Thévenin equivalent circuit models consisting of a resistance, capacitance, and power source. Voltage and resistance are measured during battery discharge at a constant 2 A direct current (DC). The experimental output results of the resistor-capacitor (RC) circuit are calculated and compared to the simulation results over the time of discharge. Both single and triple RC circuits are utilized in the experiment, with the triple RC circuit model demonstrating less error than the single RC circuit when compared to the simulated data. This study suggests that the output voltage of a Li-ion battery generally obeys a simulated Thévenin equivalent circuit model composed of the multiple RC elements under constant current discharge.

THEORETICAL BACKGROUND

The internal electrochemical reaction in a conventional two-terminal battery can be explained by a simple equivalent circuit model. Among equivalent circuit models, the Thévenin equivalent circuit model adequately applies to the operation of lithium-ion batteries[6,7] and consists of a standard parallel resistor-capacitor circuit (RC branch) and an internal resistor



The battery output voltage was 3.7 V and its total current capacity was 2 ampere hours (Ah). The extracted charge and SOC current were calculated using the following equations:

$$Q_e=\int_0^t I(\tau)d\tau, \qquad (1)$$

$$SOC=1-\frac{Q_e}{C_q}, \qquad (2)$$

where the Q_e is the extracted charge calculated based on the Coulomb count, and C_q is the total capacity of the battery.[10] Models for 1 RC branch and 3 RC branches were designed based on Thévenin equivalent circuit. By contrast, simulated values of the voltage, resistance, and capacitance in relation to the SOC were estimated with Matlab Simscape™.[11] Schematics block diagrams for the simulation are presented in Figure 2 and 3. Figure 4 depicts a flowchart used to estimate the parameters based on the data obtained from the current-voltage discharge experiment. The experimental data were then compared to the simulated estimates by fitting the experimental results based on the equivalent circuit models using nonlinear regression curves.

UNIT III
BATTERY SOC AND SOH ESTIMATION & HARMONICS MITIGATION

How does lithium-ion cell health degrade?

The loss of mobile ions reduces the maximum capacity that can be achieved by the battery. Battery lifetime can be diminished when the electrode structure is damaged through structural disordering. Structural disorder can arise during cycling as a result of the movement of Li-ions in and out of the electrodes.

A battery charge cycle refers to the complete drainage and recharge of a battery: draining a battery to 0% and recharging it to 100% is equal to one battery charge cycle. A charge cycle can also be completed by using 50% of the battery, recharging it to 100% and then repeating this procedure.

The more cycles a battery has completed the more it degrades, reducing its lifespan.

The explanation for this comes from the chemical make-up of a LiB, as there are an array of chemical mechanisms by which these batteries degrade.

One instance is the loss of mobile Li-ions in the battery. These are often lost through side reactions that occur with the electrolyte to form compounds which ‘trap’ free lithium, reducing the number of Li-ions that can shuttle between the electrodes. The loss of mobile ions reduces the maximum capacity that can be achieved by the battery.

Battery lifetime can be diminished when the electrode structure is damaged through structural disordering. Structural disorder can arise during cycling as a result of the movement of Li-ions in and out of the electrodes.

This can reduce the number of Li-ions the electrode can accept into its structure, depleting the LiB capacity.

Total-least-squares battery-cell capacity estimates:

Most of todays electric vehicles (EVs) employ lithium-ion (Li-ion) batteries due to their high volumetric energy densities. Battery packs typically consist of many cells in series or parallel configuration to provide the desired operating voltage and capacity. It is of high importance for EV designers to estimate the full usable capacity of the battery cells over time in order to maximize the efficiency and minimize the cost of

the battery system. Accurate cell capacity estimation is key to predict the total amount of energy stored in a battery pack of an EV, the electric range and the charging time over the lifetime of the vehicle.

One of the key indicators of the lifetime of a battery cell is termed state-of-health (SoH). The two aspects that have been widely used to quantify SoH are the cell total capacity and the equivalent series resistance (ESR) [1]. Cell total capacity is defined as the amount of charge that can be extracted from a cell while it is brought from a fully charged state to a fully discharged state [2]. Capacity is measured in ampere-

hours (Ah) and decays slowly due to several ageing processes that takes place inside the cell. SoH of a battery cell can be monitored with battery management systems (BMSs). Numerous methods can be found in the literature for cell capacity estimation. The most common is based on the rela- tionship between the open circuit voltage (OCV) versus state- of-charge (SoC) for a given type of battery. Although, the

variation of the OCV-SoC curve is small over the lifetime of the battery, this technique is relatively sensitive to voltage measurements because of the flat curve of OCV slope versus

SoC [3]. Advanced methods based on incremental capacity analysis (ICA) have been used to monitor SoH based on OCV data [4]. Although useful information can be extracted from the IC curve, the OCV model and voltage measurement noise limit the accuracy of capacity estimation over time. Different types of combined SoC/SoH methods (joint, dual) [1], [5], [6] have been employed for cell capacity estimation. The joint estimation can potentially provide relatively better

estimates but they heavily rely on the accuracy of the battery model, more importantly; because state and capacity estima- tion is coupled, instability/lack of convergence might occur. The dual estimation method tries to reduce the complexity of the algorithms but does not change the impact of the model

in the accuracy of the estimation [7]. A very common method to tackle the problem of errors associated with capacity estimates is to ensure that the SoC estimates are as accurate as possible. One approach is to have relaxation periods before and after each test so that the cell reaches an equilibrium state and the model-based SoC estimation is reasonably accurate. In [8] the impact of the voltage error measurement in SoC is minimized by estimating SoC in two different times after a relaxation period. Although this approach will minimize to a large extend the error in the SoC estimates, it puts constraints on how the capacity is estimated while the current sensor error remains. Estimating cell total capacity is challenging, because both variables used to estimate capacity cannot be measured per- fectly. SoC estimation introduce a certain error depending on the accuracy of the model and the performance of the EKF while current integration will introduce error from the current sensor that accumulates over time. Although some promising theoretical analysis and results exist in the literature for capacity estimation based on different least squares approachesN in [2], it remains to be seen if these methods can be used with the same success using experimental data taken from Li-ion battery cells. Most of todays electric vehicles (EVs) employ lithium-ion (Li-ion) batteries due to their high volumetric energy densities. Battery packs typically consist of many cells in series or parallel configuration to provide the desired operating voltage and capacity. It is of high importance for EV designers to estimate the full usable capacity of the battery cells over time in order to maximize the efficiency and minimize the cost of the battery system. Accurate cell capacity estimation is key to predict the total amount of energy stored in a battery pack of an EV, the electric range and the charging time over the lifetime of the vehicle. One of the key indicators of the lifetime of a battery cell is termed state-of-health (SoH). The two aspects that have been widely used to quantify SoH are the cell total capacity and

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sensor that accumulates over time.

A battery's capacity is an important indicator of its state of health and determines the maximum cruising range of electric vehicles. It is also a crucial piece of information for helping improve state of charge (SOC) estimation, health prognosis, and other related tasks in the battery management system (BMS). In this paper, we propose an improved recursive total least squares approach to online capacity estimation, which is based on the constrained Rayleigh quotient in terms of battery capacity. This approach accounts for errors in both the SOC and accumulated current measurements not traditionally considered in the battery capacity model to give an unbiased estimation. Moreover, the forgetting factor, updated by minimizing the Rayleigh quotient of the capacity estimation model, is applied to track the changes in the model and get a more precise estimation of the capacity. Finally, the performance of the proposed algorithm is validated via simulation and experimental studies on lithium-iron phosphate batteries. The estimation results show that the proposed algorithm improves capacity estimation accuracy.

3. Improved Recursive Total Least Squares Algorithm with Variable Forgetting Factor

3.1. Recursive Total Least Squares with Variable Forgetting Factor (VFF-RTLS)

From the capacity model in (3), we can see that there are errors in both the model input and output. Therefore, this section proposes a constrained Rayleigh quotient-based RTLS algorithm with a variable forgetting factor for the capacity estimation of $LiFePO_4$ batteries.

To facilitate the presentation of the proposed algorithm, the autocorrelation matrix of the capacity model input $\overline{x}(k)$ is defined as follows:

$$R = E\left[\overline{x}(k)\,\overline{x}^T(k)\right]. \tag{4}$$

The augmented input vector is defined as $x^a = [\overline{x}(k), \overline{y}(k)]$, and its autocorrelation matrix can be represented as follows:

$$R^a = E\left[x^a(k)\,x^{a\prime}(k)\right] = \begin{bmatrix} R & b \\ b & c \end{bmatrix}, \tag{5}$$

where $b = E[\overline{x}(k)\overline{y}(k)]$ and $c = E[\overline{y}(k)\overline{y}(k)]$.

The stochastic quantities R^a can be computed via an iteration formula with a forgetting factor μ $(0 < \mu \leq 1)$, as follows:

$$\begin{aligned} R^a(k) &= \begin{bmatrix} R(k) & b(k) \\ b^T(k) & c(k) \end{bmatrix} \\ &= \mu R^a(k-1) + x^a(k)\,x^{a'}(k). \end{aligned} \tag{6}$$